MATH 102 – HOMEWORK ASSIGNMENT 1

Due Friday, October 6th, 2017 at the beginning of the lecture.
Handwritten hand-ins only.

Exercise 1 (4 points)**.**
Let $\alpha \in \mathbb{R}$ be a real number and let $A \in \mathbb{R}^{m \times n}$ be a matrix with entries $a_{ij} \in \mathbb{R}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$.
Prove the following statement:
If $\alpha \cdot A = 0$, then $\alpha = 0$ or $A = 0$.

Exercise 2 (1+1+1+1 points)**.**
For each of the following pairs A and B of matrices, write down whether their product $A \cdot B$ is defined. For each product that is defined, write down the calculation of the respective product.

(a)

$$A = \begin{pmatrix} 5 \end{pmatrix}, \quad B = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 5 & 7 \\ 1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 \\ -1 & -8 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 3 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(d)

$$A = \begin{pmatrix} 0 & 2 & 5 \\ 4 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 8 \\ 0 & -2 \\ -3 & -1 \end{pmatrix}$$

(e)

$$A = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 6 & 0 \end{pmatrix}$$

Exercise 3 (2+2 points)**.**

(a) Compute the following sum and write down your computation:

$$1 + 2 + 3 + \cdots + 32.$$

(b) Prove the following statement: for each natural number $n \in \mathbb{N}$ we have

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Exercise 4 (4 points)**.**
Solve the following system of linear equations via Gaussian elimination:

$$\begin{pmatrix} 2 & 1 & -1 & 2 \\ 4 & 5 & -3 & 6 \\ -2 & 5 & -2 & 6 \\ 4 & 11 & -4 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \\ 4 \\ 2 \end{pmatrix}.$$

Exercise 5 (4 points)**.**
Solve the following system of linear equations and write down your computation:

$$\begin{pmatrix} 2 & 1 & 2 & -4 & 1 \\ 4 & 2 & 4 & -9 & 2 \\ -4 & 2 & -1 & 1 & -3 \\ 4 & 2 & 1 & -1 & 3 \\ 0 & 1 & 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ 4 \\ 4 \\ 33 \end{pmatrix}.$$