# MATH 102 – HOMEWORK ASSIGNMENT 4

*Due Friday, October 27th, 2017 before the lecture.*
*Handwritten submissions only.*

**Exercise 1** (1+1+1+1 points).
Bring the following complex numbers into the form $z = a + b\mathbf{i}$.

$$z_1 = \frac{2\mathbf{i}}{3+4\mathbf{i}} + \frac{\mathbf{i}}{3-4\mathbf{i}}, \quad z_2 = \overline{\overline{\left(\frac{5+3\mathbf{i}}{1+4\mathbf{i}}\right)} + \overline{2\mathbf{i}-7}}\,,$$

$$z_3 = \overline{2\mathbf{i} \cdot \overline{2\mathbf{i} \cdot \overline{2\mathbf{i}}}} \quad z_4 = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\mathbf{i}}{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\mathbf{i}}$$

**Exercise 2** (4 points).
Let $a, b, c, d \in \mathbb{R}$.

(1) Show that $(a + b\mathbf{i})^2 = c + d\mathbf{i}$ if and only if $a^2 - b^2 = c$ and $2ab = d$.
(2) Next, suppose that $c$ and $d$ are given; what are the possible values of $a$ and $b$ then?

**Exercise 3** (1+3 points).
Suppose we have $n + 1$ different real numbers $x_0, x_1, \ldots, x_n, x_n \in \mathbb{R}$. The $(n + 1) \times (n + 1)$ matrix

$$V = \begin{pmatrix} x_0^0 & x_0^1 & \ldots & x_0^n \\ x_1^0 & x_1^1 & \ldots & x_1^n \\ \vdots & \vdots & & \vdots \\ x_n^0 & x_n^1 & \ldots & x_n^n \end{pmatrix}$$

is called *Vandermonde matrix*.

(1) Write down the Vandermonde matrix for the triple of numbers $x_0 = 1$, $x_1 = 2$, and $x_2 = 3$.
(2) Let $c, y \in \mathbb{R}^{n+1}$. Show that $Vc = y$ implies that the polynomial

$$p(x) = c_0 + c_1 x^1 + \cdots + c_n x^n$$

satisfies $p(x_i) = y_i$ for $0 \le i \le n$.

**Exercise 4** (4 points).
Let $k \in \mathbb{N}$. Describe how the following expressions depend on $k$:

$$A_k := \mathbf{i} + \mathbf{i}^2 + \cdots + \mathbf{i}^k,$$

$$B_k := \mathbf{i} - \mathbf{i}^2 + \mathbf{i}^3 - \mathbf{i}^4 + \mathbf{i}^5 + \cdots + \mathbf{i}^k,$$

$$C_k := \mathbf{i} + \mathbf{i}^2 - \mathbf{i}^3 - \mathbf{i}^4 + \mathbf{i}^5 + \cdots + \mathbf{i}^k,$$

$$D_k := \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\mathbf{i}\right)^k$$