MATH 102 – HOMEWORK ASSIGNMENT 6

Due Monday, November 13th, 2017 before the lecture.
Handwritten submissions only.

Exercise 1 (2+2 points)**.**
In the lecture we have discussed that every permutation can be written as the composition of transpositions.

(1) Describe how every permutation $\pi \in \Pi(1, n)$ can be written as the composition of transpositions.
(2) Let $n \in \mathbb{N}$ and let $1 \leq i, j \leq n$. Describe how the transposition τ_{ij} can be written as the composition of transpositions that swap only adjacent elements, i.e., transpositions of the form $\tau_{k-1,k}$.

Exercise 2 (2+2 points)**.**
In the lecture we have discussed that every permutation can be written as the composition of transpositions.

(1) Consider the permutation $\pi \in \Pi(1, 7)$ defined by
$$(\pi(1), \pi(2), \pi(3), \pi(4), \pi(5), \pi(6), \pi(7)) = (2, 4, 6, 3, 1, 7, 5)\,.$$
Write this permutation as the product of transpositions
(2) Determine the inverse permutation of π as above. Write it again as the product of transpositions.

Exercise 3 (2+2 points)**.**
Consider a general complex-valued 2×2 matrix A_λ that depends on a parameter $\lambda \in \mathbb{C}$:
$$A_\lambda = \begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix},$$
where $a, b, c, d \in \mathbb{R}$.

(1) For which values $\lambda \in \mathbb{C}$ do we have $\det(A_\lambda) = 0$?
(2) Let $\lambda \in \mathbb{C}$ such that $\det(A_\lambda) = 0$. Compute the matrix–vector product $A_0 \cdot v$ where
$$v = \begin{pmatrix} b \\ \lambda - a \end{pmatrix}$$
and show that $A_0 \cdot v = \lambda v$.

Exercise 4 (4 points)**.**
Let $A, B, C \in \mathbb{R}^{n \times n}$ be matrices of size $n \times n$. Let $O \in \mathbb{R}^{n \times n}$ be the zero matrix. Show that for the determinant of the block matrix
$$\begin{pmatrix} A & B \\ O & C \end{pmatrix} \in \mathbb{R}^{2n \times 2n}$$
we have the identity
$$\det \begin{pmatrix} A & B \\ O & C \end{pmatrix} = \det(A) \cdot \det(C)$$
Hint: Use the representation of determinants in terms of permutations.