Math 102 – Fall Quarter 2017 – Midterm II

Full name: ______________________________

Student ID: ______________________________

Instructions:

(1) Please print your full name and your student ID.

(2) Using cheatsheets, calculators, books, or phones is **not** allowed.

(3) You have 50 minutes to complete the test.

(4) Show your work.

Problem	Points
1	
2	
3	
4	
5	
6	
Σ	

Problem 1 (10 points).
Answer the following questions by checking the right box:

(1) Is the $n \times n$ identity matrix Id_n the matrix with all entries 1?

☐ Yes ☐ No

(2) Let $A \in \mathbb{R}^{2\times 2}$. Is the function $f : \mathbb{R}^2 \to \mathbb{R}^2$, $x \mapsto Ax$ linear?

☐ Yes ☐ No

(3) What is the value of the addition $3 + 7$ in the finite field $\mathbb{F}_7$?

☐ 4 ☐ -3 ☐ 3

(4) Let k be an integer. For which value of k is the polynomial function $x \mapsto x^k$ linear?

☐ 0 ☐ 1 ☐ 2

(5) What is the integer k such that for any $\alpha \in \mathbb{R}$ and $A \in \mathbb{R}^{3\times 3}$ we have $\det(\alpha A) = \alpha^k \det(A)$?

☐ -1 ☐ 3 ☐ 1

(6) Does the formula $\det(A + B) = \det(A) + \det(B)$ hold for all 2×2 matrices A and B?

☐ Yes ☐ No

(7) Does the formula $\det(AB) = \det(A)\det(B)$ hold for all 2×2 matrices A and B?

☐ Yes ☐ No

(8) Let $z \in \mathbb{C}$ be a complex number. What is the absolute value of the complex conjugate $\overline{z}$?

☐ $|z|$ ☐ $-|z|$ ☐ $\dfrac{1}{|z|}$

(9) Let $A \in \mathbb{R}^{3\times 3}$ be invertible. What is the value of $\det(A^{-1})$?

☐ $\det(A)^{-1}$ ☐ $\det(A)$ ☐ $\det(A)^3$

(10) What is the determinant of the $n \times n$ identity matrix Id_n?

☐ n ☐ $(-1)^n$ ☐ 1

Problem 2 (8 points).
Compute the determinants of the following matrices.

(a) $$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & -1 & 6 \\ 4 & 2 & 1 \end{pmatrix}$$

(b) $$B = \begin{pmatrix} 3+2i & -6 \\ -i & 3i \end{pmatrix}$$

(c) $$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

(d) $$D = \begin{pmatrix} 3 & 1 & 2 & -1 \\ 0 & 6 & 3 & 1 \\ -6 & -2 & -4 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Problem 3 (8 points)**.**
Consider the following matrix $A \in \mathbb{R}^{3\times 3}$ and the vector $b \in \mathbb{R}^3$:

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

(a) Compute the classical adjoint $\text{adj}(A) \in \mathbb{R}^{3\times 3}$ of the matrix A.
(b) Compute the determinant of A.
(c) Compute the inverse $A^{-1} \in \mathbb{R}^{3\times 3}$ of A.
(d) Find $x \in \mathbb{R}^3$ such that $Ax = b$.

Problem 4 (6 points).
Consider the permutation $\pi \in \Pi(1,5)$ where

$$\pi(1) = 2, \quad \pi(2) = 4, \quad \pi(3) = 3, \quad \pi(4) = 5, \quad \pi(5) = 1.$$

(a) Write π as a product of transpositions. You may use the notation $\tau_{i,j}$ to denote the transposition swapping i and j.
(b) What is the sign of the permutation π?
(c) Write the transposition $\tau_{2,5}$ as a product of adjacent transpositions, i.e. transpositions of the form $\tau_{i,i+1}$.

Problem 5 (8 points).
Consider the following matrix $A \in \mathbb{R}^{3\times 3}$ and $b \in \mathbb{R}^3$:

$$A = \begin{pmatrix} 4 & -1 & -1 \\ 2 & 0 & 1 \\ 2 & 3 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

(a) Compute the determinant of A.
(b) For $i = 1, 2, 3$ we let A_i denote the matrix obtained by replacing the ith column of A with the column vector b. Compute the determinants of A_1, A_2, A_3.
(c) Find $x \in \mathbb{R}^3$ such that $Ax = b$.

Problem 6 (4 points)**.**
Let $n \geq 2$ be an integer and let $A \in \mathbb{R}^{n \times n}$ be invertible. Prove the following formula:

$$\det(\operatorname{adj}(A)) = \det(A)^{n-1}.$$