HW 1

1) Let $\alpha \in \mathbb{R}$, $A \in M_{m\times n}(\mathbb{R})$ (m×n matrix w/ entries in $\mathbb{R}$).

Suppose $\alpha A = 0$

i.e. $$\begin{pmatrix} \alpha a_{11} & \cdots & \alpha a_{1n} \\ \vdots & & \vdots \\ \alpha a_{m1} & \cdots & \alpha a_{mn} \end{pmatrix} = 0 \iff \alpha a_{ij} = 0$$ for all $1 \le i \le m$, $1 \le j \le n$.

If $\alpha = 0$, we're done. If not, $\alpha a_{ij} = 0 \Rightarrow a_{ij} = 0$ for all $1 \le i \le m$, $1 \le j \le n$.

$\Rightarrow A = 0$, the zero matrix, since all of its entries are 0.

2) If $A \in M_{m_1 \times n_1}(\mathbb{R})$, $B \in M_{m_2 \times n_2}(\mathbb{R})$, then AB defined $\iff$ $\underline{n_1 = m_2}$, & if so the resultant product AB is $m_1 \times n_2$

i) AB defined ✓ $$AB = \begin{pmatrix} 5 & 7 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ -1 & -8 \end{pmatrix} = \begin{pmatrix} 8 & -56 \\ -1 & -32 \end{pmatrix}$$

ii) $A_{2\times 3}$ $B_{2\times 2}$ $\quad 3 \ne 2$ $\quad$ <u>not defined!</u>

iii) AB defined ✓ $$AB = \begin{pmatrix} 0 & 2 & 5 \\ 4 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 8 \\ 0 & -2 \\ -3 & -1 \end{pmatrix} = \begin{pmatrix} -15 & -9 \\ 9 & 29 \\ 12 & 35 \end{pmatrix}$$

iv) AB defined ✓

$$AB = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}\begin{pmatrix} 2 & 6 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 0 \\ 6 & 18 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3) a) Note:

$$
\begin{array}{rl}
 & 1+2+3+\cdots+32 \\
+ & 32+31+30+\cdots+1 \\
\hline
 & \underbrace{33+33+\cdots+33}_{32 \text{ times}}
\end{array}
= 33(32)
$$

so $1+\cdots+32 = \boxed{\dfrac{33(32)}{2}}$

b) In general,

$$
\begin{array}{rl}
 & 1+\cdots+n \\
+ & n+\cdots+1
\end{array}
= n(n+1)
$$

$$\Rightarrow \boxed{\sum_{k=1}^{n} k = \frac{n(n+1)}{2}}$$

4)

$$
\left(\begin{array}{cccc|c}
2 & 1 & -1 & 2 & 5 \\
4 & 5 & -3 & 6 & 9 \\
-2 & 5 & -2 & 6 & 4 \\
4 & 11 & -4 & 8 & 2
\end{array}\right)
\underset{-2R_1+R_4}{\overset{\substack{-2R_1+R_2 \\ R_1+R_3}}{\sim}}
\left(\begin{array}{cccc|c}
2 & 1 & -1 & 2 & 5 \\
0 & 3 & -1 & 2 & -1 \\
0 & 6 & -3 & 8 & 9 \\
0 & 9 & -2 & 4 & -8
\end{array}\right)
$$

$$
\underset{-3R_2+R_4}{\overset{-2R_2+R_3}{\sim}}
\left(\begin{array}{cccc|c}
2 & 1 & -1 & 2 & 5 \\
0 & 3 & -1 & 2 & -1 \\
0 & 0 & -1 & 4 & 11 \\
0 & 0 & 1 & -2 & -5
\end{array}\right)
\overset{R_3+R_4}{\sim}
\left(\begin{array}{cccc|c}
2 & 1 & -1 & 2 & 5 \\
0 & 3 & -1 & 2 & -1 \\
0 & 0 & -1 & 4 & 11 \\
0 & 0 & 0 & 2 & 6
\end{array}\right)
$$

$$
\Rightarrow \quad
\begin{aligned}
x_4 &= \underline{3} \\
x_3 &= 4(3)-11 = \underline{1} \\
x_2 &= \frac{-1-6+1}{3} = \underline{-2} \\
x_1 &= \frac{5-6+1+2}{2} = \underline{1}
\end{aligned}
$$

5) $\left(\begin{array}{ccccc|c} 2 & 1 & 2 & -4 & 1 & 0 \\ 4 & 2 & 4 & -9 & 2 & -5 \\ -4 & 2 & -1 & 1 & -3 & 4 \\ 4 & 2 & 1 & -1 & 3 & 4 \\ 0 & 1 & 3 & 1 & 4 & 33 \end{array}\right)$

$$\begin{array}{c} -2R_1+R_2 \\ \sim \\ R_3+R_4 \end{array} \left(\begin{array}{ccccc|c} 2 & 1 & 2 & -4 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -5 \\ -4 & 2 & -1 & 1 & -3 & 4 \\ 0 & 4 & 0 & 0 & 0 & 8 \\ 0 & 1 & 3 & 1 & 4 & 33 \end{array}\right)$$

so right away we see $\underline{x_4 = 5}$,

$\underline{X_2 = 2}$

Now by plugging in $x_2 = 2$, $x_4 = 5$, we can reduce the remaining 3 equations to equations w/ 3 variables:

$$\begin{array}{c} \left(\begin{array}{ccc|c} 2 & 2 & 1 & 18 \\ -4 & -1 & -3 & -5 \\ 0 & 3 & 4 & 26 \end{array}\right) \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{var. } x_1 & \text{var } x_3 & \text{var } x_5 \end{array} \end{array} \overset{2R_1+R_2}{\sim} \left(\begin{array}{ccc|c} 2 & 2 & 1 & 18 \\ 0 & 3 & -1 & 31 \\ 0 & 3 & 4 & 26 \end{array}\right)$$

$$\overset{-R_2+R_3}{\sim} \left(\begin{array}{ccc|c} 2 & 2 & 1 & 18 \\ 0 & 3 & -1 & 31 \\ 0 & 0 & 5 & -5 \end{array}\right)$$

$\Rightarrow$ $\underline{x_5 = -1}$

$3x_3 = 31 - 1 = 30$, so $\underline{x_3 = 10}$

& $2x_1 = 18 + 1 - 20 = -1$, so $\underline{x_1 = -1/2}$