Please check the website for your room assignment!

General remarks:

- You will be asked to write down your name and student ID on the front page.
- You have 50 minutes to work on the exam.
- No books, calculators, phones, or cheatsheets are allowed during class.
- The first problem will take the form of multiple choice quiz: you are given answer and have to pick the right one.
- Review computing in finite fields and in the field of complex numbers.
- Homework 4, Exercise 1
- Homework 5, Exercise 1, 2
- Homework 6, Exercise 1, 2

Problem 1.
Consider the following linear systems of equations:

$$\begin{pmatrix} 3 & 2 & -1 \\ 1 & -1 & 5 \\ 2 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix},$$

$$\begin{pmatrix} 4 & 1 & 3 \\ -1 & 1 & 2 \\ -1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix},$$

$$\begin{pmatrix} 1 & -3 & 2 \\ 1 & 0 & -2 \\ 1 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix},$$

$$\begin{pmatrix} -1 & -1 & 3 \\ 2 & 1 & -4 \\ -2 & 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix},$$

$$\begin{pmatrix} 2 & -2 & 0 \\ -1 & -1 & -1 \\ -1 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -2 \end{pmatrix},$$

Recall the definition of the classical adjoint of a matrix. Now, for each of these linear systems of equations

- Compute the determinant of the system matrix.
- Compute the classical adjoint matrix.
- Using the determinant and the classical adjoint matrix, compute the inverse.
- Check whether the inverse is computed correctly by performing a matrix–matrix multiplication.
- Solve the linear system of equations using this inverse and double-check your result.
- Next, recall Cramer's rule. For each of these systems of equations, write up the determinants that appear in the auxiliary computations and use Cramer's rule.

Remark 0.1.
The inverse of the above matrices are:

$$\begin{pmatrix} -\frac{5}{2} & -\frac{1}{2} & \frac{9}{2} \\ 5 & 1 & -8 \\ \frac{3}{2} & \frac{1}{2} & -\frac{5}{2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & -\frac{7}{5} & \frac{11}{5} \\ 0 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ 0 & -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$$\begin{pmatrix} \frac{7}{3} & 2 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{2}{3} \\ \frac{4}{3} & 1 & \frac{1}{3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{2} & -\frac{1}{3} & -\frac{2}{3} \end{pmatrix}$$

Their determinants are

$$2,$$
$$-5,$$
$$12,$$
$$3,$$
$$6.$$