MATH 109 – EXERCISES

To be discussed within the first week.

Exercise 1. *Let a, b be real numbers with $b \neq 0$ and $b + 1 \neq 0$. Under which condition on a and b do we have*

$$\frac{1+a}{1+b} < \frac{a}{b}$$

Exercise 2. *Let a, b, c be real numbers with $b \neq 0$ and $b + c \neq 0$. Under which condition on a, b and c do we have*

$$\frac{c+a}{c+b} < \frac{a}{b}$$

Exercise 3. *Proof that for all non-negative real numbers a, b we have*

$$\sqrt{ab} \leq \frac{a}{2} + \frac{b}{2}$$

Exercise 4. *Show that for all non-negative real numbers a, b, c we have*

$$\sqrt[3]{abc} \leq \frac{a+b+c}{3}$$

Exercise 5. *Proof that $7^{\log_2(2t)} = 7t^{\log_2(7)}$ for all positive real numbers $t > 0$.*

Exercise 6. *Consider a cubic polynomial*

$$f(x) = ax^3 + bx^2 + cx + d$$

where a, b, c, d are real numbers. Suppose that x_0 is a real number that satisfies $f(x_0) = 0$. Find real numbers p, q, r, t such that the quadratic polynomial

$$g(x) = px^2 + qx + r$$

satisfies

$$f(x) = (x - t)g(x).$$

Express p, q, r, t in terms of a, b, c, d, x_0.

Exercise 7. *Find the roots of the cubic polynomial*

$$p(x) = 2x^3 - 10x^2 - 314x + 2002.$$

Explain how you found the answer.

Exercise 8. *For which real numbers x, y do we have*

$$6x^2 + 5y^2 + 13xy + 11x + 12y + 7 = 0$$