MATH 109 – HOMEWORK 1

Due Friday 19th. Handwritten submissions only.

Exercise 1 (2 points)
We have seen in the lecture that the logical connective $\vee$ can be expressed in terms of $\neg$ and $\wedge$. In turns out that a single logical connective is sufficient to express all other logical connectives.

Let the logical connective $\barwedge$ be defined by

$$P \barwedge Q :\iff \neg(P \wedge Q).$$

Express the logical connectives $\wedge$, $\vee$, and $\neg$ in terms of the logical connective $\barwedge$.

Exercise 2 (2 points)
Describe all pairs (x, y) of real numbers x and y that satisfy the following equation:

$$x^2 + 2x + 1 = y^2 - 6y + 9.$$

Exercise 3 (2 points)
Recall the binomial coefficient for integer parameters $0 \leq k \leq n$. Prove that

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}.$$

Exercise 4 (2 points)
Given a real number g, find all real numbers x for which

$$\big||x-1| + x - 3\big| < g.$$

Exercise 5 (2 points)
Prove the following: if x is an integer with at most three decimal digits $a_1a_2a_3$, then x is divisible by 3 if and only if $a_1 + a_2 + a_3$ is divisible by 3.

Exercise 6 (3 points)
A square number is an integer that is the square of another integer. Let x and y be two integers, each of which can be written as the sum of two square numbers. Show that the product xy can be written as the sum of two square numbers.

Exercise 7 (3 points)
Let a and b be rational numbers. Consider the polynomial

$$p(x) = ax^2 + bx + (a + b).$$

Show that if $p(0)$ and $p(-1)$ are integers, then $p(x)$ is an integer for every integer x.